CHAPTER 11

TOPICS IN PRICING AND PROFIT ANALYSIS

Questions

1. A firm that produces joint products in fixed proportions should withhold some of one product from the market if the marginal revenue of that product is negative at the point where marginal revenue for the firm (in this case represented by marginal revenue of the other product) is equal to marginal cost for the firm, represented by the marginal cost of producing the joint product.

2. A product transformation curve is used in this context to illustrate the various combinations of two products which a firm can produce with a given level of total cost. Revenue is represented by a series of isorevenue curves, which indicate the various combinations of the two products that a firm can sell to yield a given amount of revenue. The profit-maximizing level of output of each product will occur where an isorevenue curve is tangent to a product transformation curve, and the difference between total revenue and total cost at that point is greater than at any other such tangency point.

3. A transfer product is a product, produced by one division of a firm, that is used as an *input* for the product of another division of the same firm. One example of such a case is that in which the transfer product can be sold as a repair or replacement part, as well as be used in the production of another product sold by the firm. Car batteries would be an example of a product that could conceivably be such a transfer product.

4. Profit will be maximized where marginal revenue of the final product less its marginal cost (excluding the price of the transfer product) is equal to the marginal cost of producing the transfer product. We can also state this condition as the point where the net marginal revenue of the enterprise is equal to the marginal cost of the transfer product, which is equal to the net marginal cost of the enterprise. Another way of stating the profit-maximizing condition for a firm in this situation is that the firm should produce until the marginal revenue from the final product is equal to the marginal cost of the entire enterprise.

5. *Consumer's surplus* refers to the difference between what a consumer actually pays for a specific quantity of a good or service and the maximum amount that he or she would be willing to pay for that same quantity. In the case of first degree price discrimination, a different price is charged for each unit sold, and the consumer's surplus is approximated by the triangular area bounded by the consumer's demand curve, the price paid, and the vertical (price) axis. In second degree discrimination, since there are price blocks, only a part of the consumer's surplus is captured by the seller. However, the more numerous the price blocks, the greater the proportion of the surplus obtained by the seller.

6. Market segmentation is the separation of buyers into distinct markets characterized by different prices for the same, or virtually the same, product. If the cost of production is the same in each market, profit is maximized where the marginal revenue in each market is equal to the firm's marginal cost. If marginal costs are different, the firm will maximize profit by producing (and/or selling) its product where marginal revenue equals marginal cost in each separate market.

7. It would be useless to price discriminate if the price elasticities of demand were the same in the various markets (assuming marginal costs were equal) or if the firm were not able to keep consumers in the various markets segregated.

8. *Two-part pricing* is an approach that charges consumers both an access fee and a price per unit purchased. Two-part pricing is frequently found in the golf course market, where consumers pay a membership fee plus a green fee.

9. *Bundling* is an approach that enhances revenue and profit by offering consumers a "package deal" on two or more products. If two types of consumers have different demands for a pair of products, preferences between them must be *inverse* in order for bundling to succeed in increasing revenue.

10. A firm's management may wish to maximize sales subject to the constraint that an adequate level of firm profit is made, if management compensation is based on growth of sales. In this case the managers are maximizing their own individual profit, but not necessarily that of the firm. A firm's management may also choose to operate at a rate of output greater than that which would result in short-run profit maximization because they wish to discourage other firms from entering the industry. In this case the managers would take this action in an attempt to maximize the profit of the firm in the long run.

11. Multinational corporations can use transfer pricing as a means of transferring profits out of divisions located in countries with high taxes or restrictions on capital flows and accumulating them in divisions located in areas with lower tax rates or fewer restrictions on movements of capital. Obviously, such practices may have significant effects on the economies of the various countries involved.

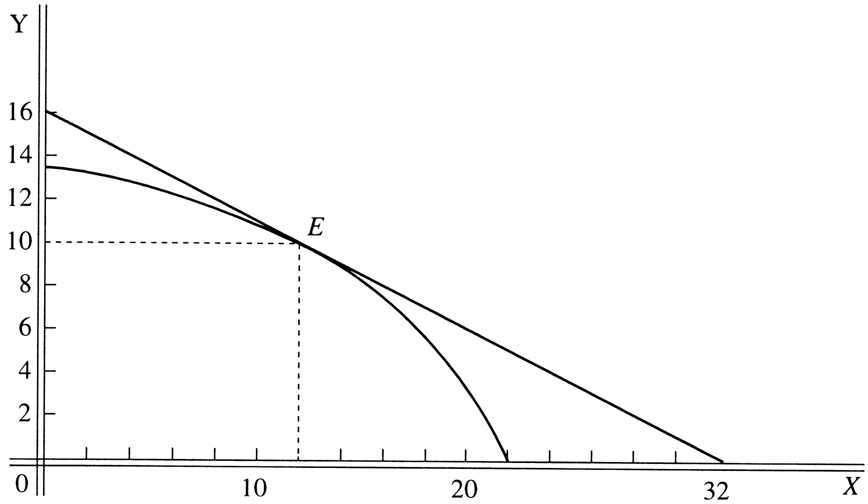
12. Price discrimination in interstate commerce is generally permitted when it can be justified on the basis of differences in grade, quality or quantity sold; differences in transportation costs; or where it involves the lowering of price in good faith to meet competition.

13. Hotels, theaters, utilities, and the airlines are some examples of firms that practice price discrimination.

14. The firm's price is determined by a "markup" percentage in one of two ways: either as a markup on price (the proportion of the selling price that represents an amount added to cost of goods sold) or as a markup on cost (the proportion of cost of goods sold that is added onto that figure to arrive at the selling price). Markup pricing is a long-run pricing strategy, as firms try to set price at a level that would allow them to achieve a certain long-run target rate of return at a particular volume of production.

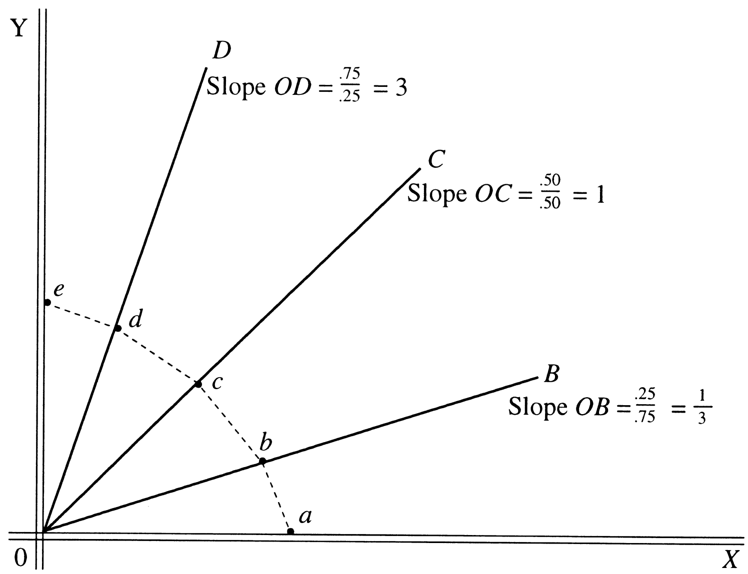
Problems

1.



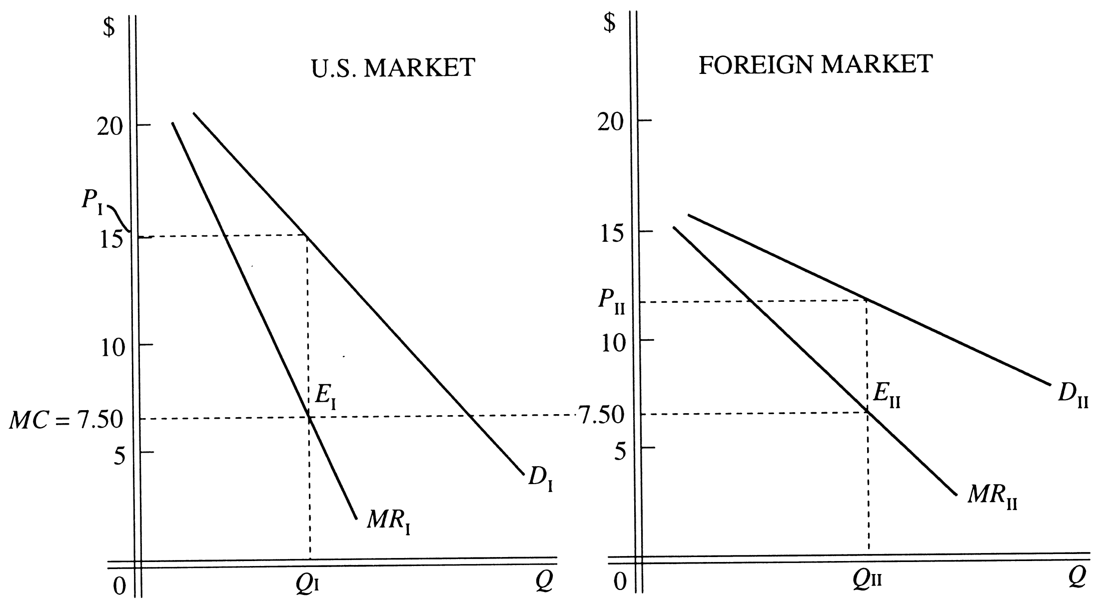
The combination of point E will yield the maximum profit ($y = 10$; $x = 11$, given the above scale). The important point is that, given $P_x = 1/2P_y$, the isorevenue line will have a slope of $-1/2$.

2.



The transformation "curve" should have the above shape because of the imperfect substitutability of inputs. Only the combinations at points *a, b, c, d*, and *e* can be produced.

3.



The condition $MR_I = MR_{II} = MC$ is satisfied at E_I and E_{II} in the preceding graph. Because of the tariff, foreigners who buy at P_{II} cannot resell in the U.S. (U.S. price = \$15.25; foreign price = \$12.50.)

4. $TR = TR_a + TR_b$

$= P_a \cdot Q_a + P_b \cdot Q_b$

$Q_a = 100 - P_a$, so $P_a = 100 - Q_a$

$Q_b = 120 - .8P_b$, so $-.8P_b = Q_b - 120$

$P_b = 150 - 1.25Q_b$

$TR = Q_a(100 - Q_a) + Q_b(150 - 1.25Q_b)$.

As long as MR_a and MR_b are both greater than zero, $Q_a = Q_b = Q_j$, and the quantity sold of both products will be the same. For profit maximization, joint marginal revenue MR_j will be equal to SMC, or

$MR_j = SMC$, where $MR_j = MR_a + MR_b$. Thus,

$250 - 4.5Q_j = 4 + 1.5Q_j$;

$6Q_j = 246$; and, therfore, $Q_j = \underline{\underline{41}}$ units.

We must check to see that the marginal revenue of each product is positive at $Q = 41$ units.

$MR_a = -2Q_a + 100 = -2(41) + 100 = \$18$

$MR_b = -2.5Q_B + 150 = -2.5(41) + 150 = \$47.50$

The firm should sell 41 units of each product.

$P_a = -41 + 100 = \underline{\underline{\$59.00}}$

$P_b = -1.25Q + 150 = -1.25(41) + 150 = \underline{\underline{\$98.75}}$

5. **a.**

PRODUCT COMBINATIONS

No. of Shifts	100% *A*	50% *A* 50% *B*	100% *B*	75% *A* 25% *B*
1	TC = \$125,000	TC = \$125,000	TC = \$125,000	TC = \$125,000
2	TC = \$265,000	TC = \$265,000	TC = \$265,000	TC = \$265,000
3	TC = \$425,000	TC = \$425,000	TC = \$425,000	TC = \$425,000

b.

PRODUCT COMBINATIONS

No. of Shifts	100% *A*	50% *A* 50% *B*	100% *B*	75% *A* 25% *B*
1	$T\pi$ = $ 75,000	$T\pi$ = $ 100,000	$T\pi$ = $150,000	$T\pi$ = $ 75,000
2	$T\pi$ = $115,000	$T\pi$ = $145,000	$T\pi$ = $235,000	$T\pi$ = $135,000
3	$T\pi$ = $ 65,000	$T\pi$ = $ 150,000	$T\pi$ = $150,000	$T\pi$ = $175,000

Profit is maximized with two shifts, producing 100% B.

6. $Q_f = 3{,}000 - 125P_f$

$MC_f = .004Q_f$

$MC_t = .008Q_t$

The firm will maximize profit relative to the final product where $NMR_f = MR_f - MC_f = P_t$.

$$-125P_f = Q_f - 3{,}000$$

$$P_f = -.008Q_f + 24$$

$$TR_f = P_f Q_f = (-.008Q_f + 24)Q_f$$

$$\frac{dTR_f}{dQ_f} = MR_f = -.016Q_f + 24$$

$$NMR_f = MR_f - MC_f = -.016Q_f + 24 - .004Q_f$$

$$NMR_f = -.020Q_f + 24$$

At the profit-maximizing quantity, $NMR_f = P_t$.

$$-.02Q_f + 24 = \$2.80$$
$$-.020Q_f = -21.20$$

a. $Q_f = \underline{\underline{1{,}060}}$ units of the final product.

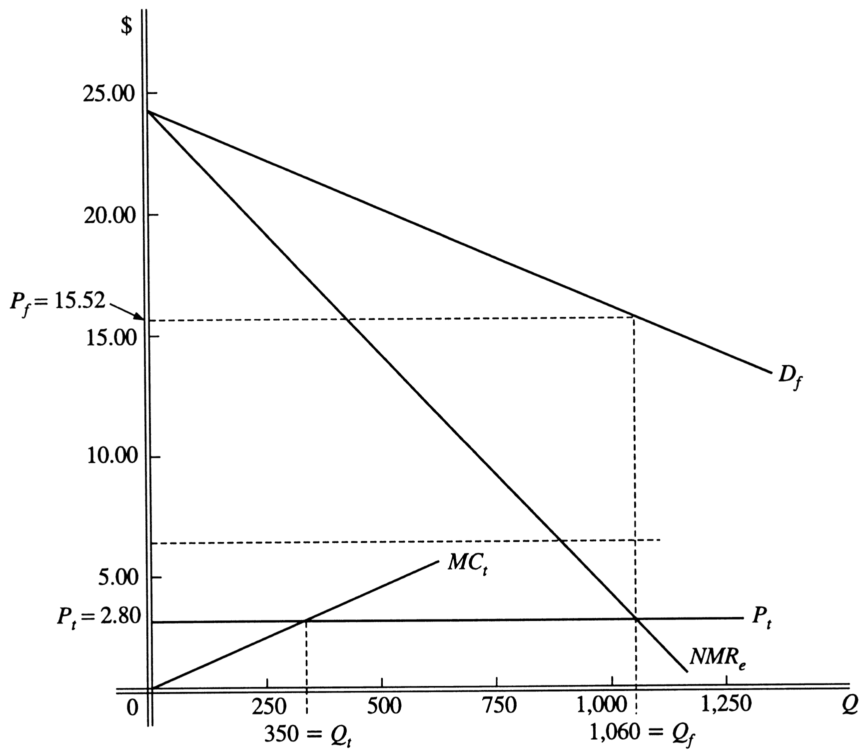
$P_f = -.008(1{,}060) + 24 = -\$8.48 + 24 = \underline{\underline{\$15.52}}$

The profit-maximizing output of the final product will be determined where $P_t = MC_t$: $\$2.80 = .008Q_t$.

b. $Q_t = \underline{\underline{350}}$ units of the transfer product should be produced.

c. No, because the profit-maximizing quantity of the components which Maxton should produce is 350 units. It needs 1,060 units for the final product and should buy 710 components externally for $2.80. The external price is cheaper than Maxton's marginal cost of production after 350 units.

d.



7. $Q_c = 10{,}000 - 2.000P_c \qquad Q_j = 6{,}000 - 1{,}000P_j$

$-2{,}000P_c = Q_c - 10{,}000 \qquad -1{,}000P_j = Q_j - 6{,}000$

$P_c = -.0005Q_c + 5 \qquad P_j = .001Q_j + 6$

$MC = \$1.20$, so the firm will maximize profit where $MR_c = MR_j = MC = \$1.20$

$TR_c = P_c(Q_c) = (-.0005Q_c + 5)Q_c = -.0005Q_c^2 + 5Q_c$

$$MR_c = \frac{dTR_c}{dQ_c} = -.001Q_c + 5$$

Where $MR_c = MC$,

$-.001Q_c + 5 = 1.20$

$-.001Q_c = -3.80$

$Q_c = \underline{\underline{3{,}800}}$ carryout orders per week

$P_c = -.0005Q_c + 5 = \underline{\underline{\$3.10}}$

$TR_j = P_j(Q_j) = (-.001Q_j + 6)Q_j = -.001Q_j^2 + 6Q_j$

$$MR_j = \frac{dTR_j}{dQ_j} = -.002Q_j + 6$$

Where $MR_j = MC$,

$-.002Q_j + 6 = 1.20$

$-.002Q_j = -4.80$

$Q_j = \underline{\underline{2{,}400}}$ eat-in servings per week

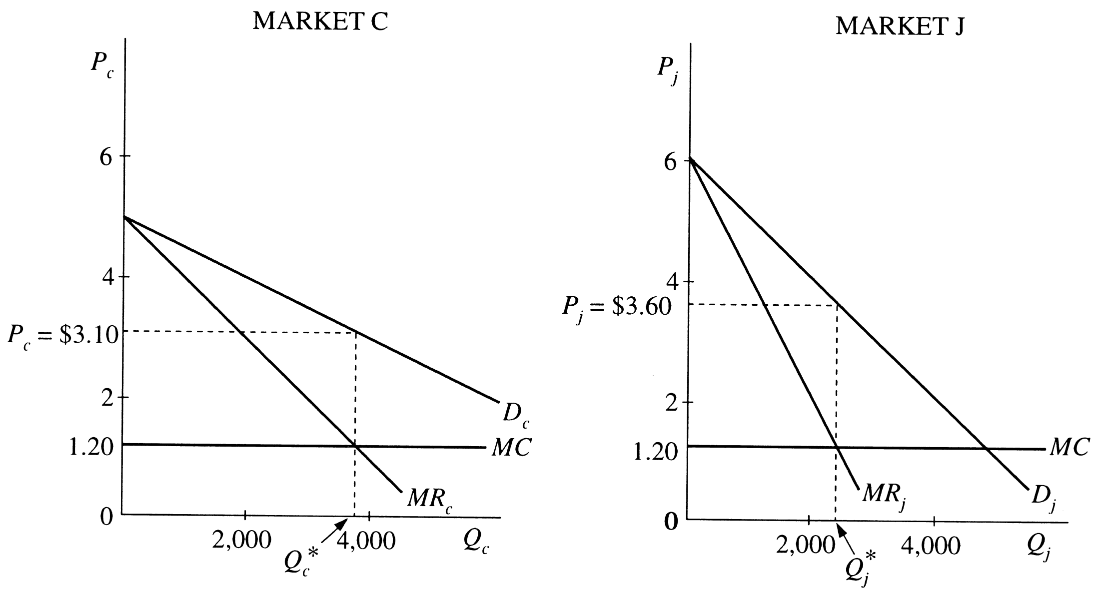
$P_j = -.001Q_j + 6 = \underline{\underline{\$3.60}}$

a. Yes

b. 3,800 carryout servings; 2,400 eat-in servings

c. P_c = \$3.10, P_j = \$3.60

d.



8. **a.** $m = -1/(E_p + 1) = -1/(-3.5 + 1) = 1/2.5 = 0.4$, or 40%.

b. $P = AVC + AVC(m) = AVC(1.4) = 18(1.4) = \underline{\underline{\$25.20}}$.

9. Considering the inverse preferences in the table and the total of the maximum amounts each type of consumer is willing to pay for the products, Smales should definitely bundle the prints and disks. Note the "Total" column below.

Type of Customer	Prints	Photodisk	Total
Type P	\$7.50	\$4.00	\$11.50
Type D	\$5.50	\$8.00	\$13.50

Without bundling, the most Smales could charge is \$5.50 for the prints (the maximum amount the *Type D* customer would pay) and \$4.00 for the photodisks (the maximum amount the *Type P* customer would pay). Both types of customers would buy both products, spending \$9.50 for the combination of prints and disks. However, as the table shows, the two products bundled can be priced at \$11.50, and both consumers will still buy both products. Smales gets an additional \$2 per customer by bundling.

10. Since $Q = 18 - P$, the AR function is $P = 18 - Q$. With marginal cost at \$2, the firm should set $P = SMC$ and charge an amount equal to the consumer's surplus for the season discount card:

$P = 18 - Q = 2$

$Q = \underline{\underline{16}}$.

The consumer's surplus will be $(1/2)(18 - 2)(Q) = 8(16) = 128$, which is the area of the triangle formed by the demand curve, the vertical axis, and the line $SMC = 2$. Thus \$128 should be the price of the discount card, and each admission should be priced at $P = SMC = \$2$.

11. **a.** The MR of shells will be zero at $Q_s = 3{,}000$. There, however, $MR_b = 180 - .04(3{,}000) = \$60$. Since the constant MC is \$50, the firm will want to continue to produce and sell beans until $MR_b = MC$, or until $180 - .04Q = 50$. Thus, $.04Q = 130$, and the production quantity of both products is $Q = 130/.04 = \underline{\underline{3{,}250}}$.

b. All of the beans should be sold, but only 3,000 units of shells should be sold, since $MR_s < 0$ beyond $Q_s = 3{,}000$. Therefore:

$P_b = 180 - .02Q_b = 180 - .02(3{,}250) = \underline{\underline{\$115}}$, and

$P_s = 7.5 - .00125Q_s = 7.5 - .00125(3{,}000) = \underline{\underline{\$3.75}}$.

c. There will be 250 excess output of shells.

d. $T\pi = TR_b + TR_s - TC = 373{,}750 + 11{,}250 - 150{,}000 - 162{,}500 = \underline{\underline{\$122{,}500}}$.

12. First, double the slope term on the $P = AR$ equation for each market to obtain the respective MRs. Then solve for the optimal quantity in each market by setting MC, which is \$40, equal to each market's MR. Thus,

$MR_r = 200 - 0.1Q_r = 40$; $0.1Q_r = 160$, and $Q_r = \underline{\underline{1{,}600}}$

$MR_a = 100 - .025Q_a = 40$; $.025Q_a = 60$, and $Q_a = \underline{\underline{2{,}400}}$.

Substituting the quantities into the price equations,

$P_r = 200 - .05(1{,}600) = 200 - 80 = \underline{\underline{\$120}}$

$P_a = 100 - .0125(2{,}400) = 100 - 30 = \underline{\underline{\$70}}$.

Total profit is $TR_r + TR_a - AVC(Q) - TFC$, so

$$T\pi = 120(1{,}600) + 70(2{,}400) - 40(4{,}000) - 150{,}000$$
$$= 192{,}000 + 168{,}000 - 160{,}000 - 150{,}000 = \underline{\underline{\$50{,}000}}.$$

C1. **a.**

$P_b = 3{,}800 - Q_b$

$MR_b = 3{,}800 - 2Q_b$

$P_z = 1{,}200 - .2Q_z$

$MR_z = 1{,}200 - .4Q_z$

$SMC = 20 + .19Q_j$, where Q_j is one unit of *b and* one unit of *z*.

One possibility is that profit will be maximized where $MR_j = SMC$.

$MR_j = MR_b + MR_z = 5{,}000 + 2.4Q_j$, and thus

$5{,}000 + 2.4Q_j = 20 + .19Q_j$,

$4{,}980 = 2.59Q_j$; $Q_j = 1{,}923$.

However, if $Q_j = 1{,}923$, then

$MR_b = 3{,}800 - 3{,}846 = -46$

$MR_z = 1{,}200 - 769.2 = 430.8$, which indicates we must reject this answer and restate the profit-maximizing condition as $MR_z = SMC$, or

$1{,}200 - .4Q_z = 20 + .19Q_z$

$.59Q_z = 1{,}180$; $Q_z = \underline{\underline{2{,}000}}$.

Thus, 2,000 of *both* products should be produced, but only 1,900 units of bubble bath should be sold, since at $Q_b = 1{,}900$, $MR_b = 0$.

b. The equilibrium prices are:

$P_b = 3{,}800 - 1{,}900 = \underline{\underline{\$1{,}900}}$ per unit;

$P_z = 1{,}200 - 400 = \underline{\underline{\$800}}$ per unit;

c. 100 units of bubble bath should be kept off the market.

C2. **a.** The first step is to find MR_d.

$$Q_d = 70{,}000 - 400P_d$$
$$-400P_d = -70{,}000 + Q_d$$
$$P_d = 175 - .0025Q_d$$
$$TR_d = P_d(Q_d) = 175Q_d - .0025Q_d^2$$
$$MR_d = 175 - .005Q_d$$

The next step is to find NMR_d, which is equal to $MR_d - MC_d$.

$$NMR_d = 175 - .005Q_d - 10 - .003Q_d$$
$$NMR_d = 165 - .008Q_d$$

The profit-maximizing quantity of disc players will be where

$$NMR_d = P_s = \$5.00$$
$$165 - .008Q_d = 5$$
$$-.008Q_d = -160$$
$$Q_d = \underline{\underline{20{,}000}} \text{ disc players.}$$

b. $$P_d = 175 - .0025Q_d$$
$$= 175 - .0025(20{,}000)$$
$$= 175 - 50$$
$$= \underline{\underline{\$125}}.$$

c. The profit-maximizing quantity of styli to produce is where $MC_s = P_s = \$5.00$

$$2 + .0002Q_s = 5$$
$$.0002Q_s = 3$$
$$Q_s = \underline{\underline{15{,}000}} \text{ styli.}$$

d. Since the styli are available at a fixed market price of $5.00 per unit, the stylus division must charge this same price when it transfers its product to the compact disc player division. There will be excess internal demand equal to 5,000 units for the styli.

C3. Given the demand curves for the two products ($Q_w = 12{,}000 - 100\,P_w$ and $Q_v = 6{,}000 - 200P_v$) we have

$P_w = 120 - 0.01Q_w$; $MR_w = 120 - 0.02Q_w$, and

$P_v = 30 - .005Q_v$; $MR_v = 30 - .01Q_v$.

$MR_v = 0$ when $Q_v = 3{,}000$, but at that output, $MR_w = 120 - 0.02(3{,}000) = 60$. Also, at that output, $MC = 4.16 + 0.012Q = 4.16 + 36 = 40.16$. Since $MR_w > MC$, the firm will wish to sell more wine. The solution occurs where $MR_w = MC$, or

$$120 - 0.02Q = 4.16 + 0.012Q$$
$$0.032Q = 115.84;\ Q = \underline{\underline{3{,}620}}.$$

The firm should sell all 3,620 units of the wine but should sell only 3,000 units of vinegar, since $MR_v < 0$ for outputs greater than 3,000. There will be 600 excess units of vinegar. From the given demand curves,

$$P_w = 120 - 0.01(3{,}620) = \underline{\underline{\$83.80}}.$$
$$P_v = 30 - 0.005(3{,}000) = \underline{\underline{\$15}}.$$

Profit will be

$$(TR - TC) = 83.80(3{,}620) + 15(3{,}000) - 100{,}000 - 4.16(3{,}620) - 0.006(3{,}620)^2$$
$$= 303{,}356 + 45{,}000 - 100{,}000 - 15{,}059.20 - 78{,}626.40 = \underline{\underline{\$154{,}670.40}}.$$

C4. **a.** The firm should produce clocks up to the point where the net marginal revenue from sale of the clocks equals the price of the springs. From the given data,

$P_c = 700 - 0.05Q_c$; $MR_c = 700 - 0.1Q_c$.

$NMR_c = (MR_c - MC_c) = 700 - 0.1Q_c - 23 - 0.12Q_c = 677 - 0.22Q_c$.

Next $NMR_c = P_s$ where $677 - 0.22Q_c = 28$, and $0.22Q_c = 649$; $Q_c = \underline{\underline{2{,}950}}$.

$P_c = 700 - 0.05(2{,}950) = \underline{\underline{\$552.50}}$.

b. Boing should produce mainsprings up to the point where $MC_s = MR_s$, or $4 + 0.005Q_s = 28$. Thus, $.005Q_s = 24$ and $Q_s = 4{,}800$. $P_s = \$28$, the market price.

c. Since Gongalong needs only 2,950 springs, the difference between Boing's production and that amount, or $4{,}800 - 2{,}950$, means that 1,850 springs will be sold to other buyers. Clearly, Gongalong will not purchase any springs from other firms.

C5. Since Peixe Louco will find that the *MR* of oil will be zero at $Q_o = 3{,}500$, it should compare the *MR* of fish to the joint *MC* at that quantity to see whether production and sale of more fish is warranted.

At $Q = 3{,}500$:

$MR_f = 100 - .02Q_f = 100 - 70 = 30$

$MC = 4.06 + .006Q = 4.06 + 21 = 25.06$.

Since $MR_f > MC$ expansion of output to the level where $MR_f = MC$ will maximize profit:

$100 - .02Q = 4.06 + .006Q$; $.026Q = 95.94$; $Q = \underline{\underline{3{,}690}}$.

The above quantity of fish should be sold. However, only 3,500 of oil should be sold. Thus, the prices are:

$P_f = 100 - .01Q_f = 100 - .01(3{,}690) = \underline{\underline{\$63.10}}$

$P_o = 35 - .005Q_o = 35 - .005(3{,}500) = \underline{\underline{\$17.50}}$.

Last, profit is:

$$\begin{aligned} T\pi &= 63.10(3{,}690) + 17.50(3{,}500) - 100{,}000 - 4.06(3{,}690) - .003\,(3{,}690)^2 \\ &= 232{,}839 + 61{,}250 - 100{,}000 - 14{,}981.40 - 40{,}848.30 \\ &= \underline{\underline{\$138{,}259.30}}. \end{aligned}$$

There is excess codfish oil of $3{,}690 - 3{,}500 = \underline{\underline{190}}$.

C6. **a.** From the cost function, $MC = 10$. From the given demand curves, $P_f = 49.2 - .05Q_f$ and $MR_f = 49.2 - .1Q_f$, while $P_c = 41.4 - .02Q_c$ and $MR_c = 41.4 - .04Q_c$. Setting the constant *MC* equal to *MR* in each market will yield:

$49.2 - .1Q_f = 10$; $Q_f = \underline{\underline{392}}$

$41.4 - .04Q_c = 10$; $Q_c = \underline{\underline{785}}$.

b. Substitution of the respective quantity values into the price form of the demand curves:

$P_f = 49.2 - .05(392) = \underline{\underline{\$29.60}}$

$P_c = 41.4 - .02(785) = \underline{\underline{\$25.70}}$.

c. $T\pi = 29.60(392) + 25.70(785) - 12{,}000 - 10(1{,}177) = \underline{\underline{\$8{,}007.70}}$.

C7. **a.** From the given data, it follows that

$P_g = 217.2 - 0.05Q_g$; $TR_g = 217.2Q_g - 0.05Q_g^2$

$P_c = 116 - 0.02Q_c$; $TR_c = 116Q_c - 0.02Q_g^2$.

Total revenue for the hotel will be the sum of the above two *TR* functions. To obtain its total cost, substitute $(Q_g + Q_c)$ for Q in the given cost function. Then, to maximize profit with discrimination, maximize the following:

$$\begin{aligned} T\pi &= 217.20Q_g - 0.05Q_g^2 + 116Q_c - 0.02Q_g^2 - 112{,}500 \\ &\quad - 40Q_g - 40Q_c - 0.02Q_gQ_c - 0.01Q_g^2 - 0.01Q_c^2. \\ &= 177.2Q_g - 0.06Q_g^2 + 76Q_c - 0.03Q_c^2 - 112{,}500 - 0.02Q_gQ_c. \end{aligned}$$

The first partial derivatives of the above with respect Q_g and Q_c will equal zero when profit is maximized.

$$\partial\pi/\partial Q_g = 177.20 - 0.12Q_g - 0.02Q_c = 0,$$

$$\partial\pi/\partial Q_c = 76 - 0.02Q_g - 0.06Q_c = 0.$$

Multiplying the first equation above by -3 and adding it to the second yields

$0.34Q_g = 455.6$; $Q_g = \underline{\underline{1{,}340}}$.

The value of Q_g can now be substituted into either of the partials above to obtain $Q_c = \underline{\underline{820}}$.

b. Substituting the respective quantity values into the price equations for the two markets yields

$P_g = \underline{\underline{\$150.20}}$,

$P_c = \underline{\underline{\$99.60}}$.

c. The profit calculation is

$$\begin{aligned} T\pi &= 150.20(1{,}340) + 99.60(820) - 112{,}000 \\ &\quad - 40(2{,}160) - 0.01(2160)^2 \\ &= 201{,}268 + 81{,}672 - 112{,}000 - 86{,}400 - 46{,}656 \\ &= \underline{\underline{\$37{,}884}}. \end{aligned}$$

d. The profit function is now constrained by the condition that price in the general market must be the same as that in the convention market. The constraint equation is

$P_g = P_c$, or $217.20 - 0.05Q_g = 116 - 0.02Q_c$.

In zero form, $101.2 - 0.05Q_g + 0.02Q_c = 0$.

The problem now is to maximize the original profit function subject to the stated constraint. This can be done by forming the Lagrangian expression and setting its first partials equal to zero.

$$\begin{aligned} L\pi &= 177.2Q_g - 0.06Q_g^2 + 76Q_c - 0.03Q_c^2 - 112{,}500 \\ &\quad - 0.02Q_gQ_c + \lambda(101.2 - 0.05Q_g + 0.02Q_c) \end{aligned}$$

(1) $\partial L\pi/\partial Q_g = 177.20 - 0.12Q_g - 0.02Q_c - 0.05\lambda = 0$

(2) $\partial L\pi/\partial Q_c = 76 - 0.02Q_g - 0.06Q_c + 0.02\lambda = 0$

(3) $\partial L\pi/\partial \lambda = 101.20 - 0.05Q_g + 0.02Q_c = 0$

Multiplying (2) by 2.5 and adding it to (1) yields

(4) $367.2 - 0.17Q_g - 0.17Q_c = 0$.

Multiplying (3) by 8.5 and adding to (4) yields

$1{,}227.4 - 0.595Q_g = 0$

$Q_g = \underline{\underline{2{,}063}}$, and, by substitution,

$367.2 - 0.17(2{,}063) - 0.17Q_c = 0$

$0.17Q_c = 16.49$; $Q_c = \underline{\underline{97}}$

Note that the sum of the quantities $(2{,}063 + 97)$ equals that same total number of rooms rented (2,160) as under price discrimination. However, substituting the respective quantities into the original price equations of the demand curves for the two markets,

$P_g = 217.20 - 0.05(2{,}063) = \underline{\underline{\$114.05}}$

$P_c = 116 - 0.02(97) = \underline{\underline{\$114.06}}$.

The prices are effectively the same, which they must be given the constraint. (The penny difference occurs because Q_g was rounded from 2,062.8 to 2,063.) The impact on profit is dramatic. Using $P = P_g = P_c = \$114.05$,

$$T\pi = 114.05(2{,}160) - 112{,}000 - 86{,}400 - 46{,}656$$
$$= \underline{\underline{\$1{,}292}}$$

Without price discrimination, the general market price drops significantly, while there is an increase in the convention market rate. Quantity drops precipitously in the convention market and increases in the general market. As the profit decline shows, the hotel gains substantially if it is able to discriminate in pricing between the two markets.

C8. For the stated demand curves,

$$P_c = 20 - 0.4Q_c \text{ (city)}$$
$$P_r = 18 - 0.4Q_r \text{ (rural)}$$

and given no discrimination between city and rural clients, it is clear that the solution requires $P_c = P_r = P$, the green fee. Since demand of the typical rural golfer is less than that of the city one, the membership fee will be limited to the consumer surplus associated with the rural golfer. The total profit contribution will be twice this fee plus the gross profit contribution on the $Q_c + Q_r$ units of golf played. The latter is the difference between the green fee and the AVC of \$3.00 per round of golf. Setting $P_c = P_r$,

$$20 - 0.4Q_c = 18 - 0.4Q_r$$
$$0.4Q_r = 2 = 0.4Q_c$$
$$Q_c = Q_r + 5.$$

Then the total profit contribution function will be

$$T\pi_c = 2(1/2)\,(P^* - P)\,(Q_r) + (P - AVC)\,(Q_r + Q_c).$$

Thus, we state

$$T\pi_c = 2(1/2)\,(18 - 18 + 0.4Q_r)\,(Q_r) + (18 - 0.4Q_r - 3)\,(2Q_r + 5)$$
$$= 0.4Q_r^2 + (15 - 0.4Q_r)\,(2Q_r + 5)$$
$$= 0.4Q_r^2 + 75 - 0.8Q_r^2 + 28Q_r$$
$$= 0.4Q_r^2 + 28Q_r + 75.$$

Differentiation of the last item above yields marginal profit contribution, which is set equal to zero.

$$M\pi_c = 0.8Q_r + 28 = 0$$
$$Q_r = \underline{\underline{35}}.$$

Next, $Q_c = (Q_r + 5) = \underline{\underline{40}}$.

The green fee is obtained by substituting Q_r into the P_r equation or Q_c into the P_c equation, and it will be $P = \$4$. Finally, the membership fee (call it F_r) will be equal to the consumer surplus of the rural type of golfer. Assuming, as stated in the text, that the income effect of a price change is negligible, this is equal to the area of the triangle formed by the rural demand curve, the vertical axis, and the line $P = 4$, or

$$F_r = (1/2)\,(18 - 4)\,(35) = \underline{\underline{\$245}}.$$

APPENDIX 11B
Mathematics of Price Discrimination

The Trenchwich Corp. problem is solved as follows.

From the given demand curve equations, the price and total revenue functions for the two markets are

$P_{us} = 15{,}000 - .5Q_{us}$; $TR_{us} = 15{,}000Q_{us} - .5Q_{us}^2$;

$P_f = 12{,}500 - .25Q_f$; $TR_f = 12{,}500Q_f - .25Q_f^2$.

Thus the profit function is

$$\begin{aligned} T\pi &= TR_{us} + TR_f - TC_{us+f} \\ &= 15{,}000Q_{us} - .5Q_{us}^2 + 12{,}500Q_f - .25Q_f^2 - 200{,}000 \\ &\quad - 2{,}000(Q_{us} + Q_f) - .5(Q_{us} + Q_f)^2 \\ &= 13{,}000Q_{us} - Q_{us}^2 + 10{,}500Q_f - .75Q_f^2 - 200{,}000 - Q_{us}Q_f. \end{aligned}$$

a. Setting both partials of the profit function equal to zero yields

$13{,}000 - 2Q_{us} - Q_f = 0$

$10{,}500 - Q_{us} - 1.5Q_f = 0.$

Multiplying the second equation by -2 and adding it to the first will cancel Q_{us} leaving

$-8{,}000 + 2Q_f = 0$; $Q_f = \underline{\underline{4{,}000}}$.

Substituting this value into either of the equations above yields $Q_{us} = \underline{\underline{4{,}500}}$, and

$P_{us} = 15{,}000 - .5(4{,}500) = \underline{\underline{\$12{,}750}}$

$P_f = 12{,}500 - .25(4{,}000) = \underline{\underline{\$11{,}500}}$.

b. The value of profit obtained by substituting the two market quantities sold into the profit function is $\underline{\underline{\$50{,}050{,}000}}$.

c. The constraint function for $P_{us} = P_f$ is

$15{,}000 - .5Q_{us} = 12{,}500 - .25Q_f$, or

$2{,}500 - .5Q_{us} + .25Q_f = 0.$

The Lagrangian function for the constrained maximum is the profit function stated before part (a) *plus* the above constraint multiplied by the undetermined multiplier, λ, or in brief form,

$L\pi = TR - TC + \lambda(2{,}500 - .5Q_{us} + .25Q_f)$.

Taking the partials of the Lagrangian with respect to Q_{us}, Q_f, and λ yields

$13{,}000 - 2Q_{us} - Q_f - .5\lambda = 0$

$10{,}500 - Q_{us} - 1.5Q_f + .25\lambda = 0$

$2{,}500 - .5Q_{us} + .25Q_f = 0.$

Multiplying the second equation by 2 and adding it to the first will cancel λ, leaving

$34{,}000 - 4Q_{us} - 4Q_f = 0.$

Now, Q_{us} can be cancelled by multiplying the constraint equation (partial with respect to λ) by -8, leaving

$6Q_f = 14{,}000$; $Q_f = \underline{\underline{2{,}333}}$.

By substitution, $Q_{us}, = \underline{\underline{6{,}167}}$.

For these two quantities, the respective price equations yield the same price, or

$P_{us} = P_f = \underline{\underline{\$11{,}917}}$.

For the two new quantities, the profit function now gives a total profit of $\underline{\underline{\$47{,}969{,}500}}$. The difference is all on the revenue side, since the total quantity sold ($8{,}500 = Q_{us} + Q_f$) and therefore, the total cost is the same as it was with the discriminatory prices.

INTERNATIONAL CAPSULE II
Markets and Pricing Strategy in International Trade

Questions and Problems

1. **a.** Incremental profit analysis suggests that the firm should only consider incremental costs and revenues associated with a new undertaking. Thus, it should not consider existing fixed or sunk costs in making a decision about whether to sell additional output abroad. In this setting, it will be profitable for the firm to sell overseas at any price that exceeds the *AVC* of exported output.

b. The two-market price discrimination model shows that profit is maximized when a lower price is charged to the market with the most elastic demand. In international trade, this frequently is the export market. With a lower export than home price, the firm may be subject to complaints of dumping.

c. Since there is likely to be excess production of one of two joint products, the firm may have an opportunity to sell the excess in a foreign market. It will be rational to sell abroad any units of a product for which the foreign *MR* exceeds the domestic *MR* as well as any units for which the foreign *MR* exceeds the negative of the home disposal cost.

2. *Dumping* is selling foreign at a price below home price (sometimes below home production cost). It is profitable for the firm as long as the foreign price is greater than *AVC*. Domestic producers are likely to view dumping as unfair competition, and most countries have laws that protect domestic industries from dumping. Dumping is a useful strategy for the firm but must be employed cautiously to avoid legal difficulties.

3. An *export trading company* is an umbrella organization that handles foreign sales for a group of firms. Japanese export trading companies have been very successful in their efforts to market their country's output in the U.S. and many other markets. Because of their success, the U.S. (1982) revamped its export cartel law (Webb-Pomerene Act, 1918) to allow more firms and financial institutions to participate in export trading companies. Under the new law it is possible for U.S. firms to take a cooperative, rather than a rivalrous stance toward export markets, since they can qualify for partial exemption from the U.S. antitrust laws.

4. Given that $TC = 800{,}000 + 120Q$, then $MC = 120$.

a. Set $MR = MC$ in each market.
In the home market,

$P_h = 500 - .05Q_h$,

$MR_h, = 500 - .1Q_h = 120; 380 = .1Q_h; Q_h = \underline{\underline{3{,}800}}$.

$P_h = 500 - .05Q_h, = 500 - 190 = \underline{\underline{310}}$.

In the foreign market,

$P_f = 600 - .08Q_f$

$MR_f = 600 - .16Q_f = 120; 480 = .16Q_f; Q_f = \underline{\underline{3{,}000}}$.

$P_f = 600 - .08Q_f = 600 - 240 = \underline{\underline{\$360}}$.

b. With the tariff of \$10 per unit, $MC = (MC + \text{tariff}) = 130$, assuming the firm pays the tariff and then markets the goods within the foreign country. Thus, to maximize profit,

$MR_f = 600 - .16Q_f = 130; 470 = .16Q_f; Q_f = \underline{\underline{2{,}938}}$

$P_f = 600 - .08(2{,}938) = 600 - 235.04 = \underline{\underline{\$364.96}}$.

An alternative solution would occur if the firm simply sells to importers who pay the tariff. Then, the price received by the firm is \$10 less than the demand price at every quantity, or $P_f = 600 - .08Q_f - 10 = 590 - .08Q_f$. Thus, $MR_f = 590 - .16Q_f$. Setting this equal to MC not including the tariff yields

$MR_f = 590 - .16Q_f = 120; 470 = .16Q_f; Q_f = \underline{\underline{2{,}938}}$.

$P_f = 590 - .08(2{,}938) = 590 - 235.04 = \underline{\underline{\$354.96}}$.

Note that the quantity here agrees with that calculated above. However, the price does not. The first price, \$364.96, is that paid by the purchasers; the second, \$354.96, is that received by the firm. They differ by the amount of the tariff, \$10. As theory predicts, the price to purchasers has increased by less than the amount of the tariff (\$364.96 − \$360.00) = \$4.96. The seller's price fell by (\$360.00 − \$354.96) = \$5.04. The sum of the buyer's price increase and the seller's price decrease is \$10, the amount of the tariff.

5. The rule that the marginal cost of the transfer product will be the transfer price consistent with profit maximization for the firm is often cast aside in international trade. The reasons vary, but they depend on such issues as tariffs, income taxation, restrictions on remittances of profits, and the locus of international financing.