CHAPTER 6

COST OF PRODUCTION

Chapter Outline

Questions

1. Historical costs are costs of the firm for which explicit payment has been made sometime in the past or for which the firm is committed in the future. Generally speaking, historical costs and accounting costs are the same. Opportunity costs, or implicit costs, are costs which, while they do not involve actual payment by a firm to factors of production, do represent costs to the firm in the sense that in order to use certain inputs in the production process, opportunities for the firm to use them elsewhere must be abandoned. Economic costs of a firm are equal to explicit costs plus opportunity costs; they are equal to private costs in the sense that we use the term in this book. Social costs of a firm are equal to the private costs of a firm plus any additional costs imposed on society by the firm for which it does not pay.

2. The firm should recognize the value of its resources in alternative uses. Some examples of opportunity costs would be an implicit salary for an owner-manager's time, an implicit rental expense on a building owned by the firm, and an implicit interest expense on capital (financial) invested in the firm.

3. A least cost combination is a combination of inputs that will enable a firm to produce a given level of output at the lowest possible cost. By finding the cost associated with each of these least cost combinations of inputs, the long-run total cost curve for the firm can be derived.

4. In the long run, none of the firm's inputs are fixed. In the short run, at least one of the inputs is fixed; therefore, the firm has fixed costs. The costs of a firm may be higher for a particular level of output in the short run than in the long run because of the presence of fixed costs.

5. The long-run average cost curve is an "envelope" curve of the short-run average cost curves.

6. Because total fixed cost does not change in the short run.

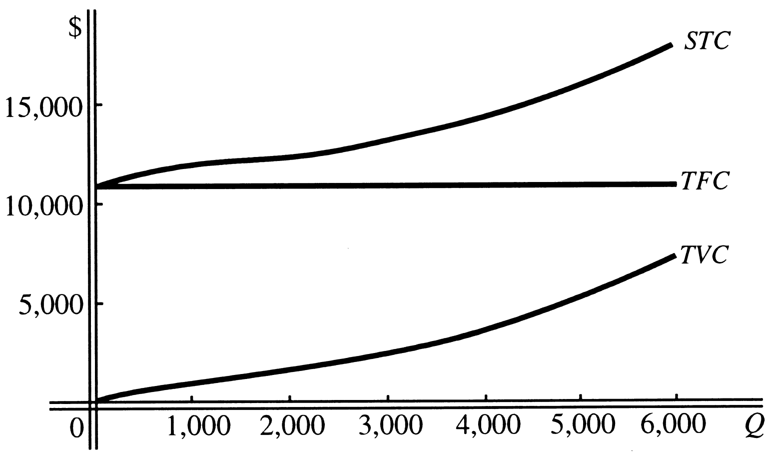
7. When the firm has increasing returns to scale, the long-run average cost curve is downsloping. When the firm has constant returns to scale, the long-run average cost curve has a zero slope. When the firm has decreasing returns to scale, the long-run average cost curve has a positive slope.

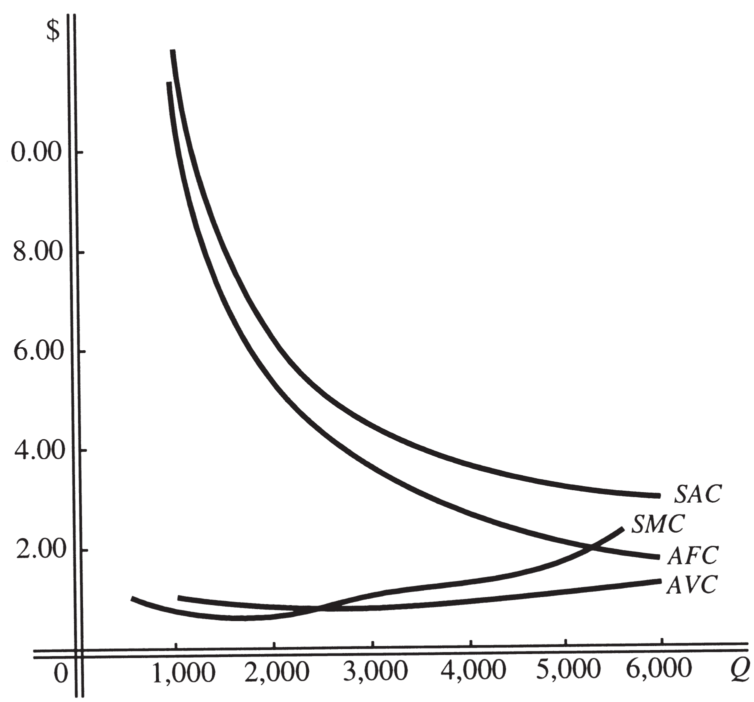
Problems

1. a.

Q	STC	TFC	TVC	SAC	AFC	AVC	Arc SMC
0	$10,800	$10,800	0	–	–	–	
1,000	11,800	10,800	1,000	$11.80	$10.80	$1.00	$1.00
2,000	12,400	10,800	1,600	6.20	5.40	.80	.60
3,000	13,200	10,800	2,400	4.40	3.60	.80	.80
4,000	14,400	10,800	3,600	3.60	2.70	.90	1.20
5,000	15,800	10,800	5,000	3.16	2.16	1.00	1.40
6,000	18,000	10,800	7,200	3.00	1.80	1.20	2.20

b.

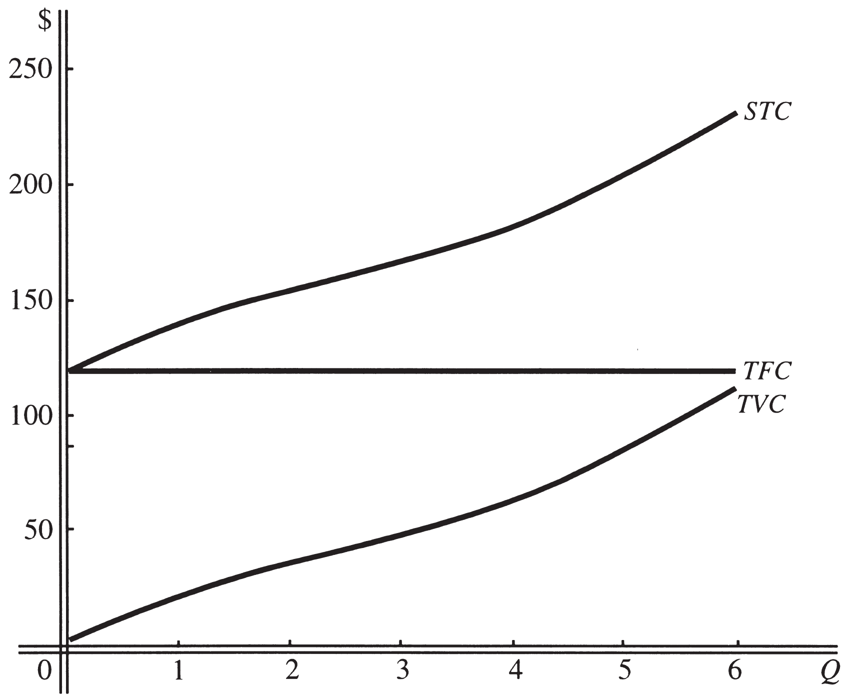


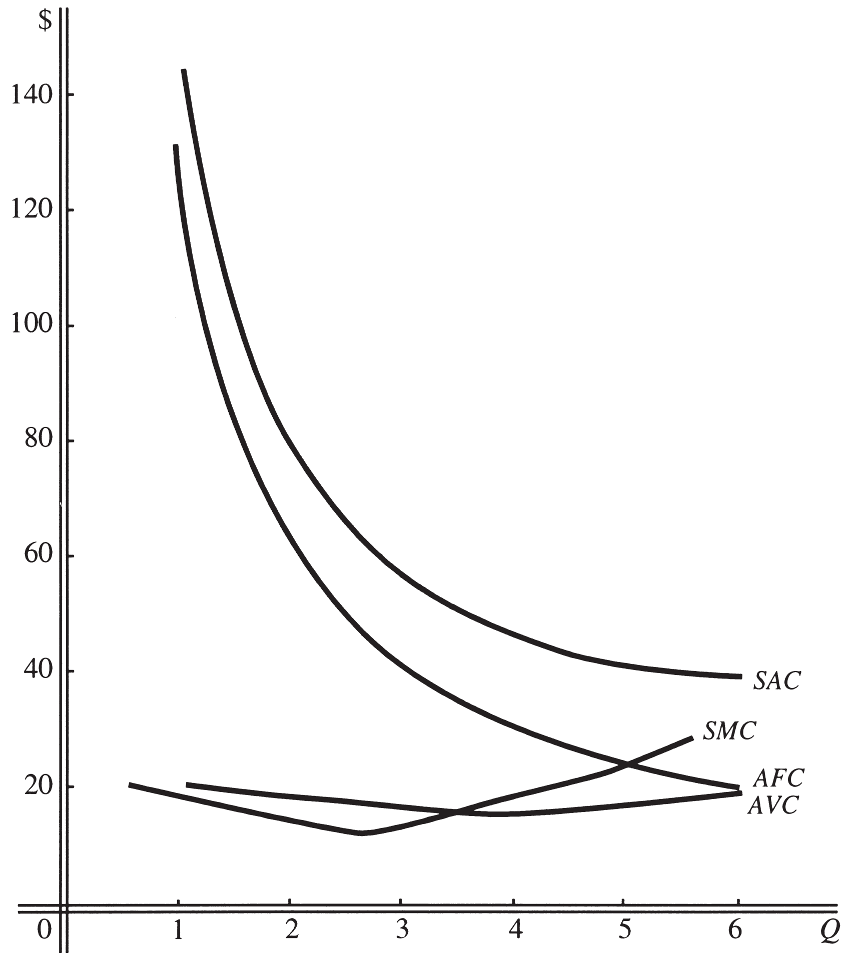


2. a.

Q	*STC*	*TFC*	*TVC*	*SAC*	*AFC*	*AVC*	Arc *SMC*
0	120	120	0	–	–	–	
1	140	120	20	140	120	20	20
2	156	120	36	78	60	18	16
3	168	120	48	56	40	16	12
4	184	120	64	46	30	16	16
5	205	120	85	41	24	17	21
6	234	120	114	39	20	19	29

b.





3. **a.** $900

b. long-run average cost = $3.00; it is consistent with point H'

c. 400

d. $700

e. $3.50

4.

Input *a* (Units)	*Q* (Output in Units)	Arc MP_a	Arc *SMC*	*AVC*	*SAC*
0	0			Undefined	Undefined
1	5	5	$10.00	$10.00	$30.00
2	20	15	3.33	5.00	10.00
3	40	20	2.50	3.75	6.25
4	55	15	3.33	3.64	5.45
5	65	10	5.00	3.85	5.39

5. **a.**

SMC	MP_L	*Q*	*L*	*TVC*	*SAC*
		0	0	$ 0	–
$ 5.00	20				
		40	2	200	$12.50
3.33	30				
		100	4	400	7.00
4.00	25				
		150	6	600	6.00
5.00	20				
		190	8	800	5.79
5.88	17				
		224	10	1,000	5.80
6.67	15				
		254	12	1,200	5.91
10.00	10				
		274	14	1,400	6.20
20.00	5				
		284	16	1,600	6.69

b. Between 40 and 100 units of output; between 40 and 100 units of output and 2 and 4 units of labor.

6. **a.** 8

b. 12

c. $12.00

d. $STC = P_a \cdot a + TFC = \$960 + \$4{,}800 = \$5{,}760$

$$SAC = \frac{STC}{Q} = \frac{\$5{,}760}{60} = \$96.00$$

7.

Input a (Units)	Output (Units)	AP_a	Arc MP_a	Arc *SMC*	*AVC*	*AFC*	*STC*
0	0	–			$ –	$ –	$ 672
			5.00	$ 40.00			
2	10	5.00			40.00	67.20	1,072
			9.00	22.22			
4	28	7.00			28.57	24.00	1,472
			10.00	20.00			
6	48	8.00			25.00	14.00	1,872
			4.00	50.00			
8	56	7.00			28.57	12.00	2,272
			2.00	100.00			
10	60	6.00			33.33	11.20	2,672
			1.50	133.33			
12	63	5.25			38.10	10.67	3,072

8. *Note:* Since $TVC = L(P_L)$, it follows from the given data that $P_L = 200/5 = TVC/L = 40$. Also, $TFC = STC - TVC = 600 - 200 = 400$. P_L can also be obtained using the reciprocal relation between SMC and MP_L. That is, $4.00 = \frac{1}{10} P_L$, therefore $P_L = 40$. The completed table follows.

SMC (Marginal Cost)	MP_L (Marginal Product)	L (Input)	$TP_L = QX$ (Output)	TVC (Total Variable Cost)	STC (Total Cost)
		5	100	200	600
2.00	20	10	200	400	800
0.80	50	15	450	600	1,000
2.00	20	20	550	800	1,200
4.00	10	25	600	1,000	1,400
8.00	5	30	625	1,200	1,600
10.00	4	35	645	1,400	1,800
13.33	3	40	660	1,600	2,000

a. (i) $AFC = 400/200 = \underline{\underline{2}}$.

(ii) $AP_L = Q/L = 600/25 = \underline{\underline{24}}$.

b. Only STC will change, since none of the other data have any fixed cost component.

9. **a.** Doubling inputs more than doubles output, so the function exhibits increasing returns to scale. Thus, long-run average cost will decrease as Q increases.

b. $MP_Y/P_Y = 35/14 = 2.5$, and $MP_Z/P_Z = 30/12 = 2.5$. Since the marginal product per dollar spent on each of the two inputs is equal, the combination is a least cost one.

c. Note that $TVC = Y(P_Y) = Y(14)$ and that $TFC = Z(P_Z) = 4(12) = 48$. The completed table follows.

SMC	MP_Y	Output of X	Input of Y	AP_Y	AVC	STC
		0	0	–	–	48
0.10	142	142	1	142.00	0.10	62
0.24	58	200	2	100.00	0.14	76
0.31	45	245	3	81.67	0.17	90
0.37	38	283	4	70.75	0.20	104
0.41	34	317	5	63.40	0.22	118
0.50	28	345	6	57.50	0.24	132

10. a. Note that $TVC = L(P_L)$, so $P_L = TVC/L = 160/2 = 80$. The completed table follows.

MP	*L*	*Q*	*STC*	*AFC*	*AVC*	*TVC*	*MC*
	0	0	120	–	–	0	
10							8.00
	2	20	280	6.00	8.00	160	
20							4.00
	4	60	440	2.00	5.33	320	
15							5.33
	6	90	600	1.33	5.33	480	
10							8.00
	8	110	760	1.09	5.82	640	
7.5							10.67
	10	125	920	0.96	6.40	800	

b. Both *STC* and *AFC* would change, since *TFC* would rise to 150.

11. a. Note that with $P_a = \$40$, *TVC* at $a = 6$ is \$240. Thus $TFC = STC - TVC = \$744 - 240 = \$504$ at the same point as well as at all other output levels. The completed table follows.

SMC	MP_a	Output (*Q*)	Input of *a*	AP_a	*AVC*	*STC*
		0	0	–	–	504
1.60	25					
		50	2	25.00	1.60	584
1.14	35					
		120	4	30.00	1.33	664
1.33	30					
		180	6	30.00	1.33	744
1.60	25					
		230	8	28.75	1.39	824
2.00	20					
		270	10	27.00	1.48	904
2.67	15					
		300	12	25.00	1.60	984

b. $AFC = \$504/180 = \$2.80$

c. Since $TFC = b(P_b)$, $P_b = TFC/b$. In this case, $P_b = \$504/12 = \$42$

12. The purpose of this problem is to demonstrate that *SAC* is a U-shaped curve even when *AVC* is an upward-sloping straight line. This is what happens when marginal cost rises linearly (total cost is a quadratic).

a. From the equation $AVC = 10 + 4Q$, it is obvious that *AVC* will be a straight, upward-sloping line. Its intercept on the dollar axis will be 10, and at $Q = 15$, *AVC* will be 70.

b. *AFC* will be a rectangular hyperbola, as always. Given that $TFC = 100$, *AFC* will equal 100 when $Q = 1$ and will fall to $\frac{100}{15} = 6.67$ at $Q = 15$.

c. When the *AVC* and *AFC* curves are added together, the result will be an *SAC* curve that has a minimum. The minimum *SAC* will occur when $Q = 5$, and $SAC = 50$. At an output of 4, $SAC = 51$, and at an output of 6, $SAC = 50.67$. When output reaches 15, *SAC* will be 76.67.

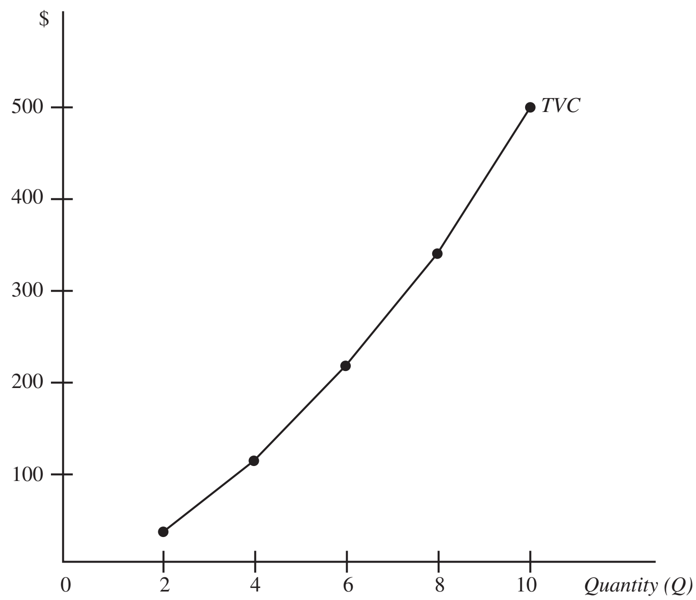
Note: While we are not requiring calculus in this problem (students can see the minimum *SAC* on their plotted curve), you can prove to yourself that the minimum *SAC* occurs at $Q = 5$ by taking $\frac{dSAC}{dQ}$ and setting that equal to zero. Since $SAC = \frac{100}{Q} + 10 + 4Q$, we have $\frac{dSAC}{dQ} = -100Q^{-2} + 4 = 0$

Thus, $4Q^2 = 100$, and $Q^2 = 25$. The positive root is $Q = 5$.

13. a. $TVC = Q(AVC) = Q(10 + 4Q) = \underline{\underline{10Q + 4Q^2}}$

b. $Q = 2, TVC = 20 + 16 = \underline{\underline{36}}$.
$Q = 4, TVC = 40 + 64 = \underline{\underline{104}}$.
$Q = 6, TVC = 60 + 144 = \underline{\underline{204}}$.
$Q = 8, TVC = 80 + 256 = \underline{\underline{336}}$.
$Q = 10, TVC = 100 + 400 = \underline{\underline{500}}$.

c.



d. The *TVC* curve has increasing slope, indicating that marginal cost is increasing as Q increases.

e. Since $AVC = a + bQ$ and $TVC = aQ - bQ^2$, the same approach used to relate linear *MR* to linear *AR* can be used to show that a linear, upward-sloping *AVC* curve will have a related marginal cost curve with *twice* the positive slope. We start with ΔTVC.

$\Delta TVC = TVC_2 - TVC_1 = aQ + a\Delta Q + bQ^2 + 2bQ\Delta Q + b\Delta Q^2 - (aQ + bQ^2) = a\Delta Q + 2bQ\Delta Q + b\Delta Q^2$

Now, to get marginal cost, divide by ΔQ,

$\Delta TVC/\Delta Q = (a\Delta Q + 2bQ\Delta Q + b\Delta Q^2)/\Delta Q = a + 2bQ + b\Delta Q = \text{SMC}$

If there is no ΔQ, which will be the case as we consider *SMC* at a single quantity produced, we get:

$SMC = a + 2bQ$

Notice that a is the intercept of both *AVC* and *SMC*. The coefficient of Q, that is, b, is the slope of *AVC*. In fact, if $AVC = 10 + 4Q$, then $SMC = 10 + 8Q$. In general, if *AVC* is a straight line with the equation $AVC = a + bQ$, *SMC* will be another straight line with the equation $SMC = a + 2bQ$.

C1. a. $SMC = \dfrac{dSTC}{dQ} = 240 - 8Q + Q^2$

$AVC = 240 - 4Q + (1/3)Q^2$

$SAC = \dfrac{1{,}000}{Q} + 240 - 4Q + (1/3)Q^2$

b. $\dfrac{dSMC}{dQ} = -8 + 2Q = 0$

$\underline{\underline{Q = 4}}$

c. $\dfrac{dAVC}{dQ} = -4 + (2/3)Q$

$\underline{\underline{Q = 6}}$

C2. a. (i) $AFC = 300/Q$

(ii) $AVC = 40 - 8Q + \frac{2}{3}Q^2$

b. $SAC = AFC + AVC = 5 + 40 - 480 + 2400 = \underline{\underline{1965}}$

c. $SMC = 40 - 16Q + 2Q^2$

d. $SMC = 40 - 320 + 800 = \underline{\underline{520}}$

e. $dAVC/dQ = -8 + \frac{4}{3}Q = 0; 4Q = 24; Q = 6$

$AVC = 40 - 48 + 24 = \underline{\underline{16}}$

C3. a. $AFC = 800/20 = \underline{\underline{40}}$

b. $SMC = 60 - 9Q + 0.45Q^2$; $dSMC/dQ = -9 + 0.9Q = 0$; $\underline{\underline{Q = 10}}$.

c. $AVC = 60 - 4.5Q + 0.15Q^2$; $dAVC/dQ = -4.5 + 0.3Q = 0$; $Q = 15$.

$AVC = 60 - 67.5 + 33.75 = \underline{\underline{26.25}}$

C4. a. $LMC = dLTC/dQ = 180 - 6Q + .06Q^2$

$LAC = LTC/Q = 180 - 3Q + .02Q^2$

b. To test for an extremum of LAC set $dLAC/dQ = 0$:

$dLAC/dQ = -3 + .04Q = 0$; $Q = 3/.04 = 75$, and $d^2LAC/dQ^2 = .04$ Thus, there is a minimum of LAC at $Q = 75$.

c. It suggests that there are variable returns to scale, since LAC first decreases but then increases.

C5. a. The foreign plant is cheaper by \$1.50 per screen.

Home plant: $SAC = STC/Q = 5{,}000/Q + 10 + .02Q$

$SAC = 5{,}000/400 + 10 + .02(400) = \underline{\underline{\$30.50}}$.

Foreign plant: $SAC_F = STC_F/Q = 6{,}400/Q + 9 + .01Q$

$SAC_F = 6{,}400/400 + 9 + .01(400) = \underline{\underline{\$29.00}}$.

b. To minimize average cost, in each case the derivative of the SAC function must equal zero.

Home plant: $dSAC/dQ = -5{,}000Q^{-2} + .02 = 0$; $Q^2 = 5.000/.02$;

$Q = 500$.

$SAC = 5{,}000/500 + 10 + .02(500) = \underline{\underline{\$30.00}}$.

Foreign plant: $dSAC_F/dQ = -6{,}400Q^{-2} + .01 = 0$; $Q^2 = 6.400/.01$;
$Q = 800$.
$SAC_F = 6{,}400/800 + 9 + .01(800) = \underline{\underline{\$25.00}}$.

c. The answer depends on the company's plans regarding future output. Presently, with the \$1,800 of allocated fixed costs removed, the average cost in the home plant for 400 units per day would be:

$SAC = 3{,}200/400 + 10 + .02(400) = \$26$.

The above is cheaper than the foreign plant at Q = 400 and would seem to be the best choice. However, for the home plant revised minimum average cost can be obtained as follows:

$dSAC/dQ = -3{,}200Q^{-2} + 02 = 0$; $Q^2 = 3{,}200/.02$; $Q = 400$.

Since the average cost minimum occurs at $Q = 400$, we know already that it will be \$26. If output is increased above 400 units per day, *SAC* will rise in the home plant. In the foreign plant, *SAC* falls until output reaches 800 units per day. Thus, foreign production could be cheaper at higher outputs. For example, if $Q = 600$, at home $SAC = 5.33 + 10 + 12 = 27.33$; but in the foreign plant, $SAC_F = 10.67 + 9 + 6 = 25.67$. An astute student may try to find where $SAC = SAC_F$ by setting the revised home *SAC* equal to the foreign one. This will solve at $Q = 518$.

C6. This problem asks about a total cost function with the following equation.

$STC = 400 + 6Q + 0.01Q^2$

a. Given that the *STC* function is a quadratic, *SMC* will be a linearly increasing function of *Q*. With marginal cost always increasing, the *STC* function will, from its intercept at $Q = 400$, rise with ever increasing slope.

b. No, as mentioned above, *SMC* will be an upward-sloping straight line, not a curve. Its equation is $SMC = 6 + 0.02Q$. Its minimum value will just be the intercept value of $SMC = 6$, but this is not an extremum of the *SMC* function.

c. Since $AVC = 6 + 0.01Q$, it is just another upward-sloping straight line. Consistent with the average-marginal relationship, it lies below the *SMC*.

d. *SAC* will have a minimum point. This occurs because of the addition of the falling (rectangular hyperbola) *AFC* curve to that of *AVC*. That is,

$$SAC = \frac{400}{Q} + 6 + 0.01Q$$

and, therefore, $\dfrac{dSAC}{dQ} = \dfrac{400}{Q^2} + 0.01$. This yields $Q^2 = 40{,}000$, and the positive root is $Q = 200$. Therefore, the minimum value of *SAC* is $SAC = \dfrac{400}{200} + 6 + 2 = \underline{\underline{10}}$.

C7. $TC = 550 + 9Q - .15Q^2 + .005Q^3$

a. $SMC = \dfrac{dSTC}{dQ} = 9 - .3Q + .015Q2$

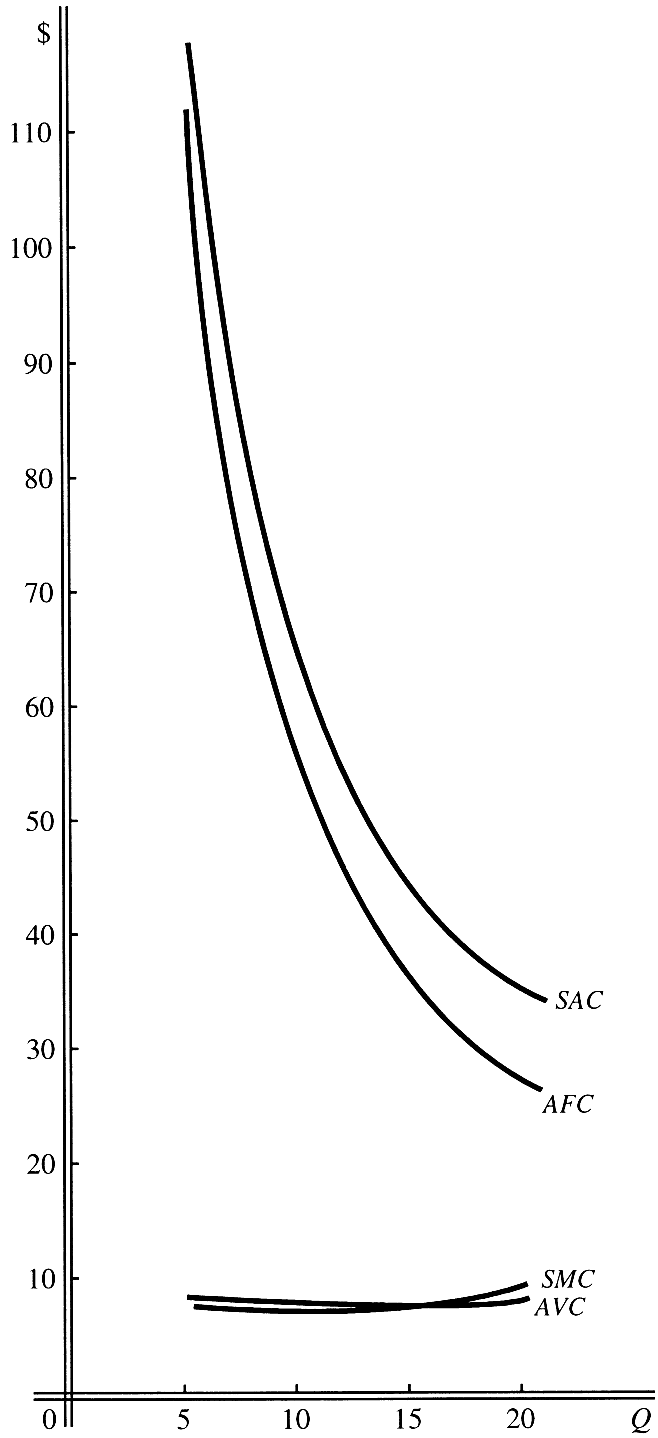
$$AVC = \frac{TVC}{Q} = \frac{9Q - .15Q^2 + .005Q^3}{Q} = 9 - .15Q + .005Q^2$$

$$SAC = \frac{STC}{Q} = \frac{550 + 9Q - .15Q^2 + .005Q^3}{Q}$$

$$= \frac{550}{Q} + 9 - .15Q + .005Q^2$$

$$AFC = \frac{TFC}{Q} = \frac{550}{Q}$$

b.



c. Minimum SMC: $\frac{dSMC}{dQ} = -.3 + .03Q = 0$

$$.03Q = .3$$

$$3Q = 30$$

$$\underline{\underline{Q = 10}}$$

Minimum AVC: $\dfrac{dAVC}{dQ} = -.15 + .01Q = 0$

$$.01Q = .15$$

$$\underline{\underline{Q = 15}}$$

Minimum AFC: at $\underline{\underline{Q = \infty}}$

d. SMC at $Q = 15$: $SMC = 9 - .3(15) + .015(15)^2$
$= 9 - 4.50 + 3.38$
$\underline{\underline{= 7.88}}$

AVC at $Q = 15$: $AVC = 9 - .15(15) + .005(15)^2$
$= 9 - 2.25 + 1.13$
$\underline{\underline{= 7.88}}$

INTERNATIONAL CAPSULE I
Some International Dimensions of Demand, Production, and Cost

Questions and Problems

1. Relative prices are the chief reason that countries can mutually gain from trading with each other. Given an exchange rate that is consistent with two-way trade, a country will export goods that are relatively cheap in its home market and import goods that are relatively expensive at home but relatively cheap abroad. The differences in relative prices from country to country frequently are based on production costs and resource endowments but may also depend on demand and other economic conditions.

2. **a.** Tablecloths are relatively cheap in England, since they require no more labor per unit than does a barrel of wine. In France, tablecloths are relatively expensive, since they require twice as much labor as a barrel of wine. Since the relative costs (prices) differ, there is a basis for two-way trade.

b. France will export wine, since wine is relatively cheap in France.

c. At 1 Euro = £1, two-way trade will not occur, since French goods are too expensive in English currency. French wine would cost £5, and French tablecloths, £10, both more than English consumers would have to pay at home. On the other hand, French consumers would want to buy *both* goods from England. For two-way trade to occur, the Euro would have to fall to something less than 4/5 of a pound sterling. For example, if the Euro were only 3/5 of a British pound, then 5 Euros, the price of a barrel of French wine, would be 0.6(5) = £3. This would make French wine cheaper for English consumers than their own wine, which is priced at £4. The English would import French wine, and the French would import English tablecloths.